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A low-carbon economic dispatch model incorporated with consumption-side emission penalty scheme



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HIGHLIGHTS

- A consumption-side emission penalty scheme is proposed.
- A low-carbon economic dispatch model based on the proposed emission penalty scheme is developed.
- The prices from the proposed dispatch model can better stimulate consumers' participation in carbon emission mitigation.
- The differentiated penalty rates help to cushion the social welfare losses.

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ABSTRACT

Due to the threats of climate change and global warming, carbon emissions are becoming a new concern during power system operation. This paper proposes a consumption-side carbon emission penalty scheme, where consumers are penalized based on their individual carbon emission responsibilities and penalty rates. Firstly, carbon emission responsibilities (CER) of consumers are determined after allocating the generators' carbon emission responsibilities to consumers through power flow tracing. Then, a low-carbon economic dispatch (LCED) model is developed with incorporation of the emission penalty scheme, in which the penalty-related cost is considered as a part of the objective function. Moreover, the consumers' differentiated penalty rates used in the LCED model are determined based on a two-level optimization model. The high-level problem determines the consumers' penalty rates which can minimize the negative impact of the penalty scheme on the social welfare while cutting the carbon emissions to a certain level. The low-level problems represent a set of LCED models to dispatch the generators' output at the given penalty rates endogenously generated within the high-level problem. Evidenced by both the theoretical analysis and simulation results, the proposed technique provides a more flexible and effective tool for the carbon emissions control compared with the traditional generation-side penalty scheme (such as carbon tax). The electricity prices derived from the proposed LCED are more stimulating for consumers to alter their electricity consumption behaviors to participate in carbon emission mitigation. Consequently, the carbon emissions can be well mitigated with less social welfare losses.

1. Introduction

Global warming caused by greenhouse gas (GHG) emission continues to cause global concern in last decades. It has become a target and constraint in future sustainable development that limits the anthropogenic GHG emissions and the corresponding global warming. As $\rm CO_2$ accounts for more than 70% of GHG emission [1], reducing the $\rm CO_2$ emission is pivotal in curbing global warming. Moreover, power systems in which fossil fuel-fired power plants play a dominate role have consistently remained a major source of $\rm CO_2$ emissions worldwide

[2]. For example, nearly half of the CO_2 emissions from coal burning is for electricity generation in China [3]. Consequently, establishment of the emission reduction targets has required the power systems to effectively reduce the CO_2 emissions. One way to reduce CO_2 emissions in power systems is introducing advanced generation technology, such as renewable energy generation and carbon capture and storage, to the generating process in the power systems [4]. Besides that, the CO_2 emission mitigation can be achieved by incorporating emission reduction schemes into the dispatch of the power systems. Considering the emission reduction schemes into the power system dispatch makes the

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Nomenclature		C_k^{gen}	generation cost function
		N_b	number of the buses
i, k	bus index (subscript)	N_l	number of the transmission lines
a, b	bus index (subscript)	N_t	number of the time periods
S	scenario index (subscript)	N_s	number of the scenarios
P_i^{dem}	real power demand of consumer at bus i (MW)	N_{ieq}	the number of inequality constraints
Q_i^{dem}	reactive power demand of consumer at bus i (MVar)	θ	phase angle of the bus voltage phase
P_k^{gen}	real power output of generating unit at bus k (MW)	V	magnitude of the bus voltage phase (kV)
E_i^{dem}	carbon emission rate of consumer at bus i (tCO ₂ /MW)	δ_{ii}	phase angle of the line's admittance
E_k^{gen}	carbon emission rate of generating unit (tCO ₂ /MW)	$ Y_{ii} $	magnitude of the line's admittance
CER_i	carbon emission responsibility of the consumer at bus i	γ	parameter of perturbation
	(tCO_2)	\boldsymbol{A}	generator to load distribution matrix
R_i	Emission penalty rate for the consumer at bus i (\$/tCO ₂)	x	variables vector
P_{i-a}^{line}	active flows on the line i-a (MW)	S	slack variables vector
P_i^{in}	total inflow to the node i (MW)	λ, μ	Lagrangian multipliers vectors
B_i^{dem}	consumers' benefit function	.,	0.0.0.

carbon emissions become a new concern during the dispatch. Therefore, it brings a better utilization of the low-carbon technologies and the consequently reduction of the system carbon emissions. Based on that, the concept of low-carbon economic dispatch, in which the carbon emission reduction become a new concern, has been proposed recently [5].

Generally, there are four schemes which are related to the lowcarbon power system dispatch [6]. The first policy is to set emission constraints for generation scheduling according to which the total quantity of emissions over a certain period should be within a specific emission limit [7,8]. The second one is to consider the emission constraints based on the emissions trading scheme (ETS) in the dispatch phase. According to the ETS, each generation company is allocated a specific allowance for emissions over the periods [9,10]. The first two policies consider the emission as a constraint during a certain period, which could be too complicated to be incorporated into the power system dispatch. The last two policies, on the contrary, are usually incorporated into the power system dispatch model. The third policy is to simultaneously minimize the generation cost and the emissions quantity [11,12]. In this way, the system dispatch model is formulated as a multi-objective optimization problem. The fourth policy is to penalize the emitters by assigning a penalty factor to (or taxing) quantity of emissions [13-15]. By introducing the penalty factors, the problem is formulated as a single objective model, in which the sum of the fuel cost and the emission-related cost is minimized. Under such circumstance, the system operator is more willing to operate the generating units with lower emissions. Thus, the system carbon emissions can be reduced.

As discussed above, penalty-based scheme could be one of the preferred emission reduction policies in the low-carbon economic dispatch since it is effective and easy to be implemented. However, traditional penalty-based scheme considers the carbon emissions from the perspective of the producers rather than the consumers. Notably, electricity is usually generated at one place with associated carbon emissions and consumed at another. Although carbon emissions are from power generation using fossil fuel, electricity production is driven by the consumers' demand. Consumers thereby should be regarded as the primary cause of carbon emissions [16]. The consensus that "consumers" rather than "producers" should be responsible for the carbon emissions during the production is growing [17]. The concept of the consumption-based carbon emissions has been proposed and utilized for transmission expansion planning in power systems [18]. Besides, the generation-side penalty scheme does not account for the consumers' specific characteristics, which are reflected by their price elasticity of demand. It should be noted that consumers could be different in their price elasticity of demand [19]. For example, the industrial consumers are usually more sensitive to the electricity prices than the residential consumers. There is no doubt that the emission penalty would be better in stimulating consumers to participate in the emission reduction if their different demand elasticity is fully considered in the design of the emission penalty scheme.

This paper proposes a consumption-side emission penalty scheme applicable in the power system dispatch, where consumers, rather than generators, are charged directly for the carbon emissions. To charge consumers directly for carbon emissions, the evaluation of consumers' carbon emission responsibilities (CERs) is necessary. Unlike the previous studies which evaluated the consumers' CERs based on the virtual 'carbon flow' analysis [16-18], this paper develops a power flow tracing-based method for determining the consumers' CERs. In the proposed technique, the generators' emissions are allocated to consumers based on how the generation is distributed among each load, and eventually the CER of each consumer can be explicitly expressed as a function depending on the system's operation variables. In this way, the consumers' penalty-related costs can be used as a part of the objective function of the low-carbon economic dispatch (LCED) model, achieving the unified optimization of system operational cost and penalty-related costs. As consumers' CERs are determined based on the power flow tracing technique which usually involves an inverse matrix [20-22], an approach is provided for dealing with the inevitable matrix inversion issue in solving the LCED model. Moreover, a two-level optimization model is proposed for determining the consumers' differentiated penalty rates used in the LCED model. The high-level problem determines the consumers' penalty rates which can minimize the negative impact of the penalty scheme on the social welfare while reducing the carbon emissions to a certain level. The low-level problems represent a set of LCED models considering the consumers' response to the electricity prices at the given penalty rates endogenously generated within the high-level problem.

Evidenced by the theoretical predictions and simulation results, the prices derived from the proposed LCED are more stimulating for consumers to alter their electricity consumption behaviors to actively participate in carbon emission mitigation. Consequently, the carbon emissions can be well mitigated with less social welfare losses. Moreover, the consumption-side emission penalty scheme, which considers the consumers' different demand elasticity, can help to cushion the negative impact of emission penalty on the social welfare. The simulation results demonstrate that the social welfare losses can be reduced if higher penalty rates are set for the consumers in the industry-sector consumers (or rather, the consumers with higher demand elasticity). The main contributions of the paper are summarized as:

- (a) A power flow tracing-based method is proposed for the assessment of consumers' carbon emission responsibilities;
- (b) A consumption-side emission penalty scheme is proposed based on which a low-carbon economic dispatch model is developed;

(c) As the power flow tracing technique involves an inverse matrix, an approach is provided for dealing with the inevitable matrix inversion issue in solving the low-carbon economic dispatch model;

(d) A two-level optimization model is proposed for determining the consumers' differentiated penalty rates used in the low-carbon economic dispatch model, in which the consumers' different demand elasticity is fully considered.

The rest of the paper is organized as follows. In Section 2, the proposed consumption-side emission penalty scheme is firstly introduced in which the consumers are charged based on their carbon emission responsibilities and pre-determined penalty rates. The power flow tracing-based method for evaluating consumers' carbon emission responsibilities is also presented in this section. In Section 3, the formulation of the low-carbon economic dispatch model considering the consumers' penalty-related costs as a part of the objective function is provided. Moreover, the two-level optimization problem determining the consumers' differentiated penalty rates is developed in this section for achieving the minimum social welfare losses when achieving the emission mitigation target. The application of the proposed technique and the algorithms for solving the optimization problems are also introduced in this section. In Section 4, the IEEE 30-bus system is modified to illustrate the effectiveness and the benefits of the proposed technique.

2. Introduction of the consumption-side emission penalty scheme

In the proposed consumption-side emission penalty scheme, consumes are charged based on their carbon emission responsibilities and the pre-determined penalty rates. Their carbon emission responsibilities, referred to as simply CERs, are defined as the carbon emissions driven by their demands. Assuming the CER of consumer i is CER $_i$, the penalty-related cost for the consumer i can be expressed as:

$$C_i^{\text{emi}}(P_i^{dem}) = R_i \cdot \text{CER}_i = R_i \cdot E_i^{dem} \cdot P_i^{dem}$$
(1)

In (1), P_i^{dem} denotes the consumer's demand (MW), E_i^{dem} denotes the consumer's carbon emission rate (tCO2/MWh) and R_i is the penalty rate for the consumer (\$/tCO2). The evaluation of CER $_i$ can be found in the following subsections. While the determination of R_i is introduced in Section 3 of the paper.

2.1. Assessment of consumers' carbon emission responsibilities

As consumers, rather than generators, are penalized for carbon emissions, it becomes necessary to answer how much carbon emissions consumers are responsible for. More specifically, it is necessary to allocate the generators' emissions to consumers. Several methods have been proposed to evaluate the carbon emission responsibility from the perspective of consumption side [16-18]. In thee references, consumers' emissions are estimated based on the concept of virtual 'carbon flow'. In the virtual carbon flow model, the carbon emissions accompanying the power delivery process is quantified with the consideration of the operational characteristics and the network features of power system and elaborately characterize the relationship between power delivery and carbon emission flow. Using the virtual 'carbon flow' method, the network of carbon emission flow from generation to demand can be built and therefore the consumers' carbon emission responsibilities are determined. Hence, the virtual 'carbon flow' method is very useful for the ex-post assessment of consumers' carbon emissions. However, the carbon flow-based method is difficult to be incorporated into the low-carbon economic dispatch formulation, as we cannot represent a consumer's carbon emission responsibility explicitly as a variable or function in the method. Therefore, this paper proposes a power flow tracing-based method for evaluating the consumers' carbon emission responsibilities. In the proposed method, consumers' carbon emission responsibilities can be expressed explicitly and hence the

corresponding emission penalty-related costs can be included in the objective function of the low-carbon economic dispatch.

In the proposed method, the generator-to-load distribution (GLD) matrix **A** is firstly through power flow tracing process. Matrix **A** is defined to describe how the generation is distributed between each of the loads. Hence, the power demand of each load can be calculated as the sum of distribution from each generator, that is:

$$P_i^{dem} = \sum_{k=1}^{N_b} \mathbf{A}_{i-k} P_k^{gen} \tag{2}$$

Based on (2), the carbon emission responsibility of consumer i can be evaluated as:

$$\begin{aligned} \text{CER}_{i} &= \triangle E^{\text{sys}}|_{P_{i}^{\text{dem}} \to P_{i}^{\text{dem}} + \triangle P_{i}^{\text{dem}}} = \sum_{k=1}^{N_{\text{b}}} \mathbf{A}_{i-k} \cdot \triangle E^{\text{sys}}|_{P_{k}^{\text{gen}} \to P_{k}^{\text{gen}} + \triangle P_{k}^{\text{gen}}} \\ &= \sum_{k=1}^{N_{\text{b}}} \mathbf{A}_{i-k} \cdot E_{k}^{\text{gen}} \end{aligned} \tag{3}$$

where $\triangle E^{\mathrm{sys}}|_{P_i^{\mathrm{dem}} \to P_i^{\mathrm{dem}} + \triangle P_i^{\mathrm{dem}}}$ denotes the additional system emissions when the demand of consumer i is infinitesimally increased by one unit; $\triangle E^{\mathrm{sys}}|_{P_k^{\mathrm{gen}} \to P_k^{\mathrm{gen}} + \triangle P_k^{\mathrm{gen}}}$ represents the additional system emissions aroused by the additional one-unit generation of generator k, corresponding to its carbon emission rate E_k^{gen} .

Therefore, CERs for all consumers can be determined given the matrix A, which is obtained by the power flow tracing technique. Moreover, as we can see from (3), a consumer's CER is the total carbon emissions from all the generators that supply the load. Hence, one effective way for a consumer to reduce the CER is shifting a part of their electricity demand to the clean distributed generation based on the renewable energies (REN).

2.2. Power flow tracing process

This subsection introduces the power flow tracing process. Among the power flow tracing methods with an analytical solution, the proportional sharing principle has been widely used [20–22]. Based on the proportional sharing principle, the topological generation distribution factor (TGDF) method has been developed. Moreover, the TGDF method is improved to avoid the matrix expansion in [22]. Therefore, the modified TGDF method proposed in [22] is used in this paper, which is briefly introduced in the following paragraphs.

Fig. 1 demonstrates the conditions at the bus i, where P_i^{gen} is the power generation, P_i^{dem} is the power demand at bus i, Ω_i is the set of buses that directly supply the bus i. It should be noted that the power generation could be a mixture of several generating units with different carbon emission rates. In this case, P_i^{gen} is the sum of the input power from different generating units, and the carbon emission rate of the input power can be calculated as the weighted average emission rate.

The symbols P_{i-a}^{line} and P_{a-i}^{line} are the active flows on the line *i-a*.

For each bus, the total nodal inflow can be defined as the sum of all the inflows from the bus i as shown in (4).

$$P_{i}^{in} = \sum_{b \in \Omega_{i}} P_{i-b}^{line} + P_{i}^{gen}, \quad i = 1, 2, \dots N_{b}$$
(4)

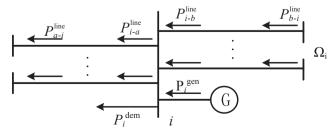


Fig. 1. Schematic diagram of the conditions at the bus i.

Moreover, additional virtual buses can be added to represent the transmission losses [8]. Since the active losses are always positive, all additional buses in the system act as the sinks. After the transmission losses are taken into the account (4), can be reformulated to obtain a matrix notation:

$$\mathbf{A} \cdot \mathbf{P} = \mathbf{G} \tag{5}$$

where P is the unknown vector of nodal flows, and G is the vector of generators' output. Due to introduction of additional buses in order to allow for transmission losses, the dimension of the distribution matrix A is enlarged to $(N_b + N_l) \times (N_b + N_l)$, where N_l is the number the transmission lines. Also, the vectors P and G are enlarged to the dimension of $(N_b + N_l)$.

Moreover, the (i, b)th element of the matrix A can be expressed as:

$$a_{ib} = \begin{cases} 1 & i = b \\ -P_{i-b}^{line}/P_b^{in} & b \in \Omega_i; i \neq b \\ 0 & b \notin \Omega_i; i \neq b \end{cases}$$

$$(6)$$

Assuming that the inverse distribution matrix $T = \mathbf{A}^{-1} \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ exists, then the i – th element of vector \mathbf{P} can be expressed as

$$P_i^{in} = \sum_{k=1}^{N_b} t_{ik} P_k^{gen}, \quad i = 1, \dots, (N_b + N_l)$$
(7)

where t_{ik} is the (i, k)th element of the matrix T. By using the proportional-sharing principle, the line power flow P_{i-j}^{line} can be presented as:

$$P_{i-j}^{line} = \frac{P_{i-j}^{line}}{P_i^{in}} P_i^{in} = \frac{P_{i-j}^{line}}{P_i^{in}} \sum_{k=1}^{N_b} t_{ik} P_k^{gen}$$
(8)

Then, the line generation distribution factor $LGDF_{ij,k}$ is defined to denote the share of the power flow on the line i-j that is produced by the generator at bus k:

$$LGDF_{ij,k} = \frac{t_{ik}P_{gen}^{gen}}{P_{g}^{gen}} \tag{9}$$

In the end, the generator-to-load distribution factor can be calculated as:

$$\mathbf{A}_{i-k} = \frac{\sum_{j \in \psi_i} LGDF_{ij,k} P_{i-j}^{line}}{P_k^{gen}} P_i^{dem}$$
(10)

3. The low-carbon economic dispatch model considering the Consumption-side emission penalty scheme

3.1. Formulation of the low-carbon economic dispatch model

The proposed LCED is applicable to the current day-ahead market, in which the independent system operator (ISO) clears the market based on the complex offers and bids submitted by generation companies and consumers. The generators' offers mainly include the fuel cost formulated as a quadratic function. The consumers' bids include hourly fixed load part and price-responsive load part. Hourly fixed loads represent consumers' base loads (CBL) which are price-taking loads satisfied at market-clearing prices [23]. A typical bid is shown as Fig. 2.

Moreover, the LCED is formulated as an optimization problem which maximizes the social welfare and subject to applicable transmission network, generation unit, and system constraints. Hence, the objective function can be expressed as:

$$\max_{P_{i}^{dem}, P_{k}^{gen}} = \sum_{t=1}^{N_{t}} \left(\sum_{i=1}^{N_{b}} B_{i}^{dem}(P_{i,t}^{dem}) - \sum_{i=1}^{N_{b}} C_{i}^{gen}(P_{i,t}^{gen}) - \sum_{i=1}^{N_{b}} R_{i} E_{i,t}^{dem} P_{i,t}^{dem} \right)$$
(11)

where

$$E_{i,t}^{dem} = \sum_{k=1}^{N_b} E_{k,t}^{gen} \mathbf{A}_{i-k} = \sum_{k=1}^{N_b} E_{k,t}^{gen} \frac{\sum_{j \in \Psi_i} LGDF_{ij,k} P_{i-j}^{line}}{P_{i,t}^{dem}}$$
(12)

In (11), the first part denotes the consumers' benefit functions embodied in their bids, the second part denotes the generators' offers and the last part represents the penalty-related costs. Moreover, as shown in (11), the decision variables of the low-carbon economic dispatch model are the scheduling of the generators' output $P_{i,t}^{gen}$ and the price-dependent loads $P_{i,t}^{dem}$. The constraints that the LCED subjects to include the power balance constraints at each bus, the power output bounds of each generator, the transmission capacity constraints, constraints limit the consumers' electricity demand, and so on.

3.2. Determination of the penalty rates for different consumers

As discussed above, the emission penalty rates R_i can be different among consumers considering the consumers' different demand elasticity. For example, a higher penalty rate can be set for the industry-sector consumers (consumers with higher demand elasticity), which contributes to the participation of those consumers in the carbon emission reduction. Hence, the regulator is supposed to set up proper penalty rates for different consumers, in order to achieve the emission reduction target with the minimum social welfare losses. For that purpose, this paper proposes a two-level optimization model determining the consumers' differentiated penalty rates used in the LCED model

The low-level problems are a set of LCED models covering different scenarios. The daily load curves describing the load level at each time period would have a significant impact on the system operation and the system emissions. Therefore, N_s scenarios representing N_s typical daily load curves are considered in the low-level problems for covering different operating situations. Each scenario represents a typical load profile. Eq. (13) presents the LCED formulation for scenario s.

$$\begin{split} \max_{\{p_{i,s,t}^{dem}, P_{i,s,t}^{gen}\}} &= \sum_{t=1}^{N_{T}} \left(\sum_{i=1}^{N_{b}} B_{i,s}^{dem} \left(P_{i,s,t}^{dem} \right) - \sum_{i=1}^{N_{b}} C_{i}^{gen} \left(P_{i,s,t}^{gen} \right) \right. \\ &- \sum_{i=1}^{N_{b}} R_{i} E_{i,s,t}^{dem} P_{i,s,t}^{dem} \\ &- \sum_{i=1}^{N_{b}} R_{i} E_{i,s,t}^{dem} P_{i,s,t}^{dem} \\ &- \sum_{i=1}^{N_{b}} E_{i,s,t}^{gen} A_{i-k} = \sum_{k=1}^{N_{b}} E_{k}^{gen} \frac{\sum_{j \in \Psi_{i}} LGDF_{ij,k} P_{i-j}^{line}}{P_{i,s,t}^{dem}} \\ &P_{i,s,t}^{dem} + \sum_{j=1}^{N_{b}} V_{i} V_{j} \mid Y_{ij} \mid \cos(\theta_{i} - \theta_{j} - \delta_{ij}) - P_{i,s,t}^{gen} = 0 \\ &Q_{i,s,t}^{dem} + \sum_{j=1}^{N_{b}} V_{i} V_{j} \mid Y_{ij} \mid \sin(\theta_{i} - \theta_{j} - \delta_{ij}) - Q_{i,s,t}^{gen} = 0 \\ &P_{i,s,t}^{dem} \leqslant P_{i}^{dem} \leqslant P_{i}^{dem} \\ &P_{i,s,t}^{gen} \leqslant P_{k}^{gen} \leqslant P_{k}^{dem} \\ &P_{i-j}^{gen} \leqslant P_{i-j}^{gen} \end{cases} \end{split}$$

As shown in (13), the objective of the LCED for scenario s is to

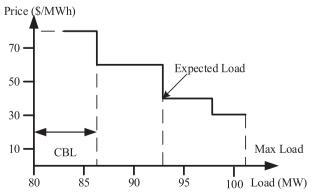


Fig. 2. Illustration of a typical demand bid

maximize the social welfare for this scenario at the given penalty rates R_i generated within the high-level problem. Moreover, the decision variables are the generator's output $P_{i,s,t}^{gen}$ and demand of the price-dependent loads $P_{i,s,t}^{den}$.

The high-level problem determines the consumers' penalty rates R_i to minimize the negative impact of the penalty scheme on the social welfare while cutting the carbon emissions to a certain level. Therefore, the high-level problem is formulated as:

$$\begin{aligned} & \max_{R_{i}} \sum_{s=1}^{N_{s}} \left\{ \sum_{t=1}^{N_{T}} \left(\sum_{i=1}^{N_{b}} B_{i,s}^{dem} \left(P_{i,s,t}^{dem} \right) - \sum_{i=1}^{N_{b}} C_{i}^{gen} \left(P_{i,s,t}^{gen} \right) \right. \\ & - \sum_{i=1}^{N_{b}} R_{i} E_{i,s,t}^{dem} P_{i,s,t}^{dem} \right) \right\} \\ & s. t. & \begin{cases} \frac{R_{i} \leqslant R_{i} \leqslant \bar{R}_{i}}{\sum_{s=1}^{N_{T}} \sum_{t=1}^{N_{D}} \sum_{i=1}^{gen} P_{i,s,t}^{gen}} \leqslant \chi \\ \left\{ P_{i,s,t}^{dem}, P_{i,s,t}^{gen} \right\} \right\} \in LP \end{aligned}$$

$$(14)$$

where χ denotes the upper limit of the total emissions.

The interactions between the high-level problem and low-level problems are shown in Fig. 3. On one hand, the low-level problems operate the LCED models at the given penalty rates R_i generated in the high-level problem to make the scheduling plan of the generators and price-dependent loads. On the other hand, the high-level problem determines the penalty rates to make sure the carbon emissions can be reduced with the minimum social welfare losses. In other word, the consumers' demand and generators' output for each scenario ($P_{i,s,t}^{dem}$, $P_{i,s,t}^{gen}$) which are necessary in the high-level problem are generated from the low-level problems.

3.3. Application and algorithm for the solution of the dispatch model

The applications of the proposed technique include the following steps: firstly, a set of representative days are selected to model the N_s scenarios; then, the two-level optimization problem described in Section 3.2 is solved to determine the penalty factors for consumers; finally, the LCED model described in Section 3.1. is launched in every operational day to achieve the low-carbon operation of the power system.

The LCED model described in Section 3.1 is a standard nonlinear constrained optimization problem, which can be solved using the interior point method (IPM) [24]. The most common approach to solving the two-level optimization problem is to replace the low-level problems

by their KKT conditions [25]. In this way, the two-level optimization problem can be written as a standard optimization problem, which can also be solved using the IPM. The process of the IPM is briefly introduced as follows while the details can be found in [26].

In the following, all the variables are represented by x. The method involves the use of a barrier function and a vector of positive slack variables s to convert the inequality constraints into equality constraints. As a consequence, the formulation can be converted into an equality constrained optimization problem:

$$\min \left[\mathbf{f}(\boldsymbol{x}) - \gamma \sum_{c=1}^{N_{\text{ine}}} \ln(\boldsymbol{s}(c)) \right]$$

$$s. \ t. \quad \boldsymbol{g}(\boldsymbol{x}) = 0$$

$$\boldsymbol{h}(\boldsymbol{x}) + \boldsymbol{s} = 0$$

$$\boldsymbol{s} > 0 \tag{15}$$

where N_{ieq} represents the number of inequality constraints and γ is the parameter of perturbation, c is the index of the inequality constraint. For a given value of γ , the Lagrange function can be formulated as:

$$L^{\gamma}(\mathbf{x}, \mathbf{s}, \lambda, \mu) = f(\mathbf{x}) + \lambda^{T} \mathbf{g}(\mathbf{x}) + \mu^{T} (\mathbf{h}(\mathbf{x}) + \mathbf{s}) - \gamma \sum_{c=1}^{N_{\text{ine}}} \ln(\mathbf{s}(c))$$
(16)

According to KKT conditions, the optimal solution of the problem must satisfy:

$$\begin{cases} \mathbf{L}_{\mathbf{x}}^{\gamma}(x, s, \lambda, \mu) = \mathbf{f}_{\mathbf{x}} + \lambda^{\mathrm{T}}\mathbf{g}_{\mathbf{x}} + \mu^{\mathrm{T}}\mathbf{h}_{\mathbf{x}} = 0 \\ \mathbf{L}_{\mathbf{s}}^{\gamma}(x, s, \lambda, \mu) = \mu^{\mathrm{T}} - \gamma \mathbf{e}^{\mathrm{T}}[\mathbf{s}]^{-1} \\ \mathbf{L}_{\lambda}^{\gamma}(x, s, \lambda, \mu) = g(\mathbf{x})^{\mathrm{T}} = 0 \\ \mathbf{L}_{\mu}^{\gamma}(x, s, \lambda, \mu) = h(\mathbf{x})^{\mathrm{T}} + \mathbf{s}^{\mathrm{T}} = 0 \end{cases}$$
(17)

where L_{χ}^{γ} , L_{s}^{γ} , L_{λ}^{γ} and L_{μ}^{γ} are the partial derivatives of Lagrange function L^{γ} with respect to x, s, λ , μ , respectively. [s] is used to denote a diagonal matrix with vector s on the diagonal and e is a vector of all ones.

Eq. (17) can be rewritten as:

$$F(x, s, \lambda, \mu) = 0 \tag{18}$$

wher

$$\mathbf{F}(\mathbf{x}, \mathbf{s}, \lambda, \mu) = \begin{bmatrix} \mathbf{L}_{\mathbf{x}}^{\mathsf{TT}} \\ [\mu]s - \gamma e \\ g(x) \\ h(x) + s \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{\mathbf{x}}^{\mathsf{T}} + \mathbf{g}_{\mathbf{x}}^{\mathsf{T}} \lambda + \mathbf{h}_{\mathbf{x}}^{\mathsf{T}} \mu \\ [\mu]s - \gamma e \\ g(x) \\ h(x) + s \end{bmatrix} = 0$$
(19)

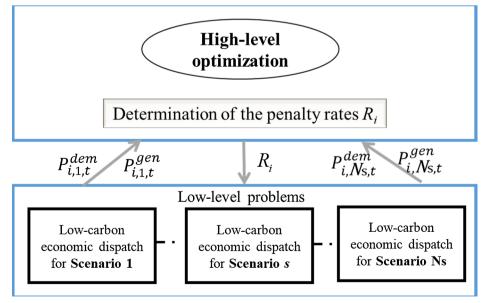


Fig. 3. Illustration of the proposed two-level optimization problem.

The optimal conditions, expressed in (19), then can be solved using Newton's iteration method.

As we can see, in the process of solving the problem, the first derivatives and second derivatives of f(x), g(x) and h(x) are utilized in each iteration. g(x) and h(x) in the proposed technique are nothing different than those of the standard economic dispatch models. However, the emission penalty component in f(x) will bring difficulty in calculating the derivatives of f(x). As discussed in the previous section, the assessment of consumers' CER is based on the power flow tracing and matrix inversion is inevitable in the power flow tracing process. In the power flow tracing method applied in this paper, matrix $T = A^{-1}$ is built to find the relationship between the nodal flows and nodal productions. Therefore, the real problem is boiled down to how to evaluate the first order and second order derivatives of T with respect to x.

The IEEE 4-bus system is used here to illustrate the formulation of matrix T, which is shown in (20).

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{p_{2-1}^{line}}{p_{2}^{dem}} & 1 & 0 & 0 \\ -\frac{p_{3-1}^{line}}{p_{1}^{dem}} & 0 & 1 & -\frac{p_{3-4}^{line}}{p_{4}^{dem}} \\ -\frac{p_{1}^{line}}{p_{1}^{dem}} & -\frac{p_{4-2}^{line}}{p_{2}^{dem}} & 0 & 1 \end{bmatrix}^{-1}$$

$$(20)$$

To the best of the authors' knowledge, it is impossible to express the elements of **T** explicitly in terms of **x**. Nevertheless, there is a theorem for the derivative of inverse matrix, that is [27]:

$$\partial \mathbf{T}/\partial x_p = \partial \mathbf{A}^{-1}/\partial x_p = -\mathbf{A}^{-1}(\partial \mathbf{A}/\partial x_p)\mathbf{A}^{-1}$$
(21)

The expression of second-order derivation can be deduced as:

$$\frac{\partial^{2} \mathbf{T}}{\partial x_{p} x_{q}} = \frac{\partial (-\mathbf{A}^{-1} \frac{\partial \mathbf{A}}{\partial x_{p}} \mathbf{A}^{-1})}{\partial x_{q}} = (\mathbf{A}^{-1} \frac{\partial \mathbf{A}}{\partial x_{q}} \mathbf{A}^{-1}) \frac{\partial \mathbf{A}}{\partial x_{p}} \mathbf{A}^{-1} - \mathbf{A}^{-1} \frac{\partial^{2} \mathbf{A}}{\partial x_{p} \partial x_{q}} \mathbf{A}^{-1} + \mathbf{A}^{-1} \frac{\partial \mathbf{A}}{\partial x_{p}} (\mathbf{A}^{-1} \frac{\partial \mathbf{A}}{\partial x_{q}} \mathbf{A}^{-1})$$
(22)

Utilizing (21) and (22), the "derivative of inverse matrix" problem can be solved easily. Hence, the obstacle of incorporating power flow tracing technique into the dispatch formulation has been removed. Moreover, it should be noted that $\partial A/\partial x_p$ for any variable x_p is a spare matrix, which allows the further simplification of (21) and (22) to reduce the computation burden.

4. Case studies

4.1. Test system introduction

An illustrative example is conducted based on the modified IEEE 30bus system [28]. The test system has six generators located at bus 1, 2, 5, 8, 11 and 13, respectively. Moreover, the generating units in the test system are classified into three groups, which are referred to simply as Clean-Unit, Gas-Units, and Coal-Units. The details about the topology of the test system, generators' parameters are shown in Supplementary Information (SI). There are 20 loads, located at bus 2-5, 7, 8, 10, 12, 14-16, 18-21, 23, 24, 26, 29 and 30, respectively. Those loads are assumed to be price-taking loads, with a benefit function of $B_i = 45 \cdot P_i^{dem}$. Besides the price-taking loads, three additional loads (Consumer 1, Consumer 2 and Consumer 3) at bus 17, 25, 27 are considered as three representative types of consumers with elastic electricity demand. It is assumed that Consumer 1, Consumer 2 and Consumer 3 represent the residential consumer, commercial consumer and industrial consumer, respectively. Moreover, their price elasticity of electricity demand is set as 0.25, 0.28 and 0.38, respectively [29]. In this section, eight scenarios are developed to cover the representative days, which involve four seasons: spring (S1 and S2), summer (S3 and S4), autumn (S5 and S6) and winter (S7 and S8). Moreover, the

weekday and weekend scenarios are also separated. The daily load curves in the eight scenarios are derived from the historical data, which are given in the SI [30–32].

To illustrate the effectiveness of the proposed technique, three cases (Case 0, Case 1 and Case 2) are modeled. In Case 0, no emission penalty is considered. In Case 1, traditional generation-side carbon tax scheme is applied, i.e. the generators are charged for the carbon emissions. In Case 2, the proposed consumption-side emission penalty scheme is incorporated into the low-carbon dispatch model. Moreover, the consumers' penalty rates are determined based on the proposed two-level optimization model. The simulation results from the three cases are analyzed in terms of the carbon emission reduction, social welfare and demand response.

4.2. Simulation results

(1). System emission and social welfare comparisons

The implementation of emission penalty scheme is expected to encourage the cleaner generating units to produce more electricity so as to reduce the system carbon emissions. However, there is no doubt that the social welfare compared with the no-penalty scenario would be reduced. The simulation results in term of carbon emissions and social welfare of the three cases are shown in Table1. In Case 1, the carbon tax is set as $10\$/tCO_2$ for all generators. In Case 2, the emission penalty rates for the Consumer1 to Consumer 3 are set as $0.9\$/tCO_2$ and $0.9\$/tCO_2$ and $0.9\$/tCO_2$ to achieve the same social welfare as Case 1, set as $0.9\$/tCO_2$ and $0.9\$/tCO_2$ to achieve the same system carbon emissions as Case 1.

From Table1 we can see that the system carbon emissions can be reduced remarkably when the emission penalty is introduced. Compared with no-penalty scenario, the system carbon emissions both in Case1 and Case2 can be decreased by more than 35%, from 356.47 tCO2/h to 223.42 tCO2/h and 223.07 tCO2/h, respectively. This is because the generators with lower emission rates are favored when the emission penalty is considered. The effect of the emission penalty in encouraging cleaner generating units to produce more electricity power is demonstrated by Fig. 4. Without the consideration of emission penalty, electricity generation from Gas-Units and Clean-Units accounts for 16.7% and 50% in the total generation, respectively. When the emission penalty scheme is implemented, the share of the Gas-units increases dramatically to 69.1% and the share of the Clean-units increases to 23.1%. Meantime, the share of Coal-units, on the contrary, decreases from 33.3% to 7.8%. It should be noted that the results in term of the generators' outputs and system emissions will not be influenced by the welfare functions for the price-taking loads. Such conclusion is verified in the Section SI.4.

The other observation from Table 1 is that the emission penalty is going to have a negative impact on the social welfare. Compared with no-penalty case, the social welfare in Case 1 decreases from 29,850 \$/h to 17,805 \$/h. In Case 2, where the proposed technique is applied, the negative impact of emission penalty on the social welfare is cushioned as shown in Table 1. In Case 2, the social welfare is 18,905 \$/h when limits the carbon emissions to 223.07 tCO₂/h, which is 6.17% higher than that in Case 1. Moreover, the system carbon emissions can be reduced to 212.87 tCO₂/h in Case 2 when the social welfare is equal to that in Case 1. In other word, the system carbon emissions can be

Table 1
Simulation results in different cases.

Cases		System carbon emission (tCO ₂ /h)	Social welfare (\$)
Case 0		356.47	29,850
Case 1		223.42	17,805
Case2	(0,9\$,21\$)	207.92	17,809
	(0,8\$,15\$)	223.07	18,905

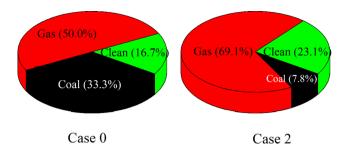


Fig. 4. Output of the generating units in Case 0 and Case 2.

 Table 2

 Consumers' demand at different capacities of the clean-unit.

Capacity (MW)	Consumer 1	Consumer2	Consumer3	Total				
Demand of the Consumers in Case 1								
100	179.01	87.64	62.53	329.18				
150	179.55	87.85	63.19	330.59				
200	180.00	88.07	63.84	331.91				
Demand of the Consumers in Case 2								
100	178.55	87.94	59.00	325.49				
150	179.02	88.14	60.32	327.48				
200	180.00	88.35	61.64	329.99				

reduced by 6.79% in Case 2 compared with Case 1 at the same social welfare losses. Hence, we can conclude that shifting the emission penalty to the consumers with higher demand elasticity contributes to reducing the social welfare losses achieving the same emission mitigation target.

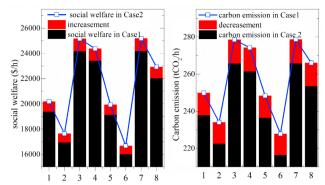
(2). Consumers' response to the penalty scheme

The proposed technique is able to encourage consumers to change their electricity consumption patterns in response to supply conditions. Specifically, the proposed technique aims to induce lower electricity consumption when the generators with high carbon emissions are dominant, and to induce higher electricity utilization while the clean generators are producing more electricity. Such effects are illustrated in Table 2. From Table 2 we can see that with the capacity of Clean-unit increasing, consumers' demand increases correspondingly. When the output of the Clean-unit is limited at 100 MW, the electricity demands of the price-dependent consumers are 325.49 MW in Case 2. When the upper output limitation of the Clean-unit increases to 150 MW, the electricity demands of the price-dependent consumers increase to 327.48 MW. The output limitation of the Clean-unit being increased to 200 MW, the price-dependent consumers' electricity demands further increase to 329.99 MW.

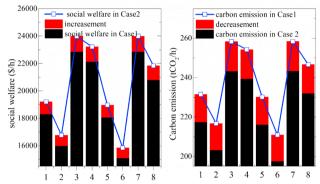
The simulation results prove that the proposed technique can encourage consumers to shift their electricity demand to the hours when the clean generators produce more electricity, which contributes to the carbon emission mitigation. Moreover, as evidenced by Table 2, the proposed technique is more effective than the traditional carbon tax scheme.

(3). Sensitivity analysis in term of the price-dependent consumers' demand elasticity

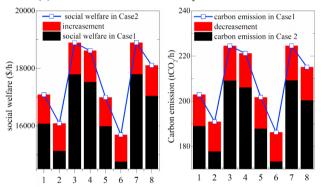
As discussed above, the proposed consumption-side emission penalty contributes to reducing more carbon emissions and cushioning the social welfare losses. Specifically, the social welfare can be increased in Case 2 compared with Case 1 achieving the same emission reduction target; additional carbon emission reduction can be achieved in Case 2 compared with Case 1 at the same social welfare level. The reason behind it is that the prices from the proposed dispatch model better stimulate consumers' participation in carbon emission mitigation.



(a) Simulation 1: demand elasticity of Consumer 3 is 0.38.



(b) Simulation 2: demand elasticity of Consumer 3 is 0.435.



(c) Simulation 3: demand elasticity of Consumer 3 is 0.655.

Fig. 5. The social welfare increases and emission reduction in Case 2.

Moreover, the greater the demand elasticity difference among consumers, more obvious the effect is. Such conclusion can be demonstrated by the following sensitivity analysis. In the following three simulations, the demand elasticity of Consumer3 is set as 0.38, 0.435 and 0.655, respectively [29].

Based on the simulation results in Fig. 5, it can be concluded that the social welfare can be increased by 4.11-6.17% using the proposed technique. Moreover, the emissions in Case 2 can be further decreased by 4.99%, 5.47% and 6.79% depending on the demand elasticity difference among consumers.

(4). Sensitivity analysis in term of the capacity adequacy

To represent the generation resource adequacy, the generation-demand capacity ratio (CR) is defined. The initial value of the CR is set as 1.5. Specifically, the total demand of the price-independent loads is 283.4 MW, the maximum demand of the price-dependent consumers is 400 MW, and the total available generation capacity is 1025 MW. In this subsection, the sensitivity analysis is conducted to test how results depend on the capacity adequacy.

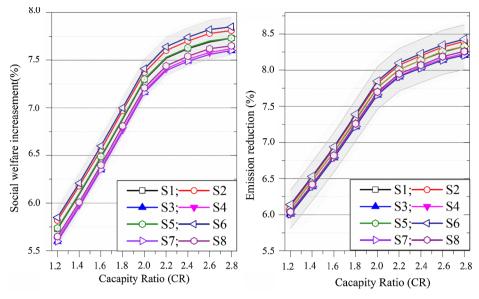


Fig. 6. The increased social welfare and additional carbon emission reduction in Case2 at different capacity ratio.

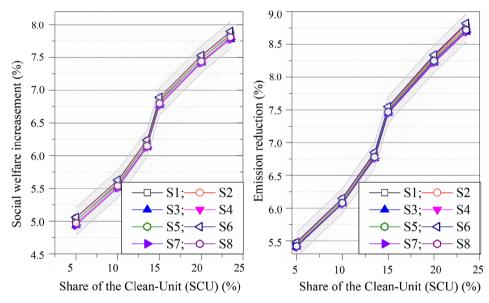


Fig. 7. The increased social welfare and additional carbon emission reduction in Case2 at different Clean-Unit level.

The simulation results are concluded in Fig. 6 while the details can be found in SI. Fig. 6 shows the increased social welfare in Case 2 compared with Case 1 when achieving the same carbon emission mitigation target and the additional carbon emission reduction in Case 2 at the same social welfare losses. As we can see from Fig. 6, the advantage of the proposed technique could be amplified with the increase of the capacity ratio. When the capacity ratio is 1.2, a 5.69% higher social welfare can be achieved in Case 2 at the same emission reduction target, and 6.06% more carbon emissions can be reduced in Case 2 at the same social welfare level. When the capacity ratio is set as 2.8, the social welfare increment in Case 2 is further increased to 7.69% while the additional carbon emission reduction is further increased to 8.30%.

(5). Sensitivity analysis in term of the share of Clean-Unit

This subsection conducts the sensitivity analysis to test how sensitive are simulation results to different energy mixes. Moreover, the different energy mixes are reflected by the different shares of the Clean-Units (SCU), i.e. share of the installed capacity of Clean-Unit in the total

generation. In the subsection, the simulations are conducted at different shares of Clean-Unit, which are 5%, 10%, 13.5%, 15%, 20%, 23.5%, respectively. The simulation results are concluded in Fig. 7 while the details can be found in SI. Fig. 7 shows the increased social welfare in Case 2 compared with Case 1 when achieving the same carbon emission mitigation target and the additional carbon emission reduction at the same social welfare losses. As we can see from Fig. 7, the advantage of the proposed technique could be amplified when the share of Clean-Unit is increased. Specifically, when the share of Clean-Unit is 5%, the social welfare in Case 2 is 4.99% higher than that in Case1 at the same emission reduction target, and 5.43% more carbon emissions can be reduced at the same social welfare level. When the share of Clean-Unit increases to 23.5%, the social welfare increment in Case2 is further increased to 7.83% while the carbon emission reduction is further increased to 8.74%. The reason behind it is that the proposed technique could do more to stimulate the utilization of the zero-emission generation. Hence, the effect is more obvious when there is more clean generation. Hence, more clean generation allows more significant advantages of the proposed technique.

5. Conclusions

In this paper, a consumption-side emission penalty scheme where consumers rather than generators are penalized for the carbon emissions is proposed. Then, a low-carbon economic dispatch model is developed with incorporation of the penalty scheme. The simulation results demonstrate that: the emission penalty is an effective tool in reducing the carbon emissions; compared with the traditional generation-side carbon tax policy, the proposed consumption-side penalty scheme is better in reducing the carbon emissions with less social welfare losses; shifting the penalty burden from the consumer with low demand elasticity to the consumers with higher demand elasticity helps to reduce the social welfare losses.

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Appendix A. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.apenergy.2019.01.108.

References

- [1] EPA, Global greenhouse gas emissions data. (available on) < https://www.epa.gov/ghgemissions/global-greenhouse-gas-emissions-data > .
- [2] Ding Y, Shao C, Yan J, Song Y, Zhang C, Guo C. Economical flexibility options for integrating fluctuating wind energy in power systems: the case of China. Appl Energy 2018;228:426–36.
- [3] Yan Q, Wang Y, Baležentis T, Sun Y, Streimikiene D. Energy-related CO2 emission in China's provincial thermal electricity generation: driving factors and possibilities for abatement. Energies 2018;11.
- [4] Brouwer AS, Broek MVD, Seebregts A, Faaij A. Operational flexibility and economics of power plants in future low-carbon power systems. Appl Energy 2015;156:107–28.
- [5] Chen D, Gong Q, Zou B, Zhang X, Zhao J. A low-carbon dispatch model in a wind power integrated system considering wind speed forecasting and energy-environmental efficiency. Energies 2012;5(4):1245–70.
- [6] Tang L, Ping C. Generation scheduling under a CO2 emission reduction policy in the deregulated market. IEEE Trans Eng Manage 2013;60(2):386–97.
- [7] Gjengedal T. Emission constrained unit-commitment (ECUC). IEEE Trans Energy Conversion 1996;11(1):132–8.
- [8] Abdul-Rahman KH, Shahidehpour SM, Aganagic M, Mokhtari S. A practical resource scheduling with OPF constraints. IEEE Trans Power Syst 1995;11(1):254–9.
- [9] Pulgar-Painemal HA. Short-term generation scheduling under a SO₂ emissions

- allowances market. Electr Power Syst Res 2005;74(2):257-65.
- [10] Kockar I, Conejo AJ, Mcdonald JR. Influence of the emissions trading scheme on generation scheduling. Int J Electr Power Energy Syst 2009;31(9):465–73.
- [11] Muslu M. Economic dispatch with environmental considerations: tradeoff curves and emission reduction rates. Electr Power Syst Res 2004;71(2):153–8.
- [12] Catalão JPS, Mariano SJPS, Mendes VMF, Ferreira LAFM. Short-term scheduling of thermal units: emission constraints and trade-off curves. Int Trans Electric Energy Syst 2008;18(1):1–14.
- [13] Cadogan JB, Eisenberg L. Sulfur oxide emissions management for electric power systems. IEEE Trans Power Apparatus Syst 2006;96(2):393–401.
- [14] Palanichamy C, Babu NS. Analytical solution for combined economic and emissions dispatch. Electr Power Syst Res 2008;78(7):1129–37.
- [15] Rahmani-Andebili M, Venayagamoorthy GK. Stochastic optimization for combined economic and emission dispatch with renewables. p. 1252–58.
- [16] Kang C, Zhou T, Chen Q, Xu Q, Xia Q, Ji Z. Carbon emission flow in networks. Sci Rep 2012;2:479.
- [17] Li B, Song Y, Hu Z. Carbon flow tracing method for assessment of demand side carbon emissions obligation. IEEE Trans Sustain Energy 2013;4(4):1100-7.
- [18] Sun Y, Kang C, Xia Q, Chen Q, Zhang N, Cheng Y. Analysis of transmission expansion planning considering consumption-based carbon emission accounting. Appl Energy 2017;193:232–42.
- [19] Pielow A, Sioshansi R, Roberts MC. Modeling short-run electricity demand with long-term growth rates and consumer price elasticity in commercial and industrial sectors. Energy 2012;46(1):533–40.
- [20] Bialek J. Topological generation and load distribution factors for supplement charge allocation in transmission open access – Discussion 1997;12(3):1185–93.
- [21] Bialek J. Allocation of transmission supplementary charge to real and reactive loads. Power Systems IEEE Transactions on 1998;13(3):749–54.
- [22] Panto's Ms, Verbic G, Gubina F. Modified topological generation and load distribution factors. IEEE Trans Power Syst 2005;20(4):1998–2005.
- [23] Wu H, Shahidehpour M, Khodayar ME. Hourly demand response in day-ahead scheduling considering generating unit ramping cost. IEEE Trans Power Syst 2013;28(3):2446–54.
- [24] Quintana VH, Torres GL, Medina-Palomo J. Interior-point methods and their applications to power systems: a classification of publications and software codes. IEEE Trans Power Syst 2000;15(1):170-6.
- [25] Ruiz C, Conejo AJ. Pool strategy of a producer with endogenous formation of locational marginal prices. IEEE Trans Power Syst 2009;24(4):1855–66.
- [26] Zimmerman RD, Murillo-Sanchez CE. Matpower 5.1-User's manual. Power Syst Eng Res Center (PSERC) 2015.
- [27] Petersen KB, Pedersen MS. The matrix cookbook. Technical University of Denmark; 2008. p. 15.
- [28] Wang C, Nehrir MH. Analytical approaches for optimal placement of distributed generation sources in power systems. IEEE Trans Power Syst 2004;19(4):2068–76.
- [29] Shu F, Hyndman RJ. The price elasticity of electricity demand in South Australia. Energy Policy 2011;39(6):3709–19.
- [30] Kang C, Zhou T, Chen Q, Wang J, Sun Y, Xia Q, et al. Carbon emission flow from generation to demand: a network-based model. IEEE Trans Smart Grid 2015;6(5):2386–94.
- [31] Wei W, Liu F, Wang J, Chen L, Mei S, Yuan T, et al. Robust environmental-economic dispatch incorporating wind power generation and carbon capture plants. Appl Energy 2016;183:674–84.
- [32] Ji Z, Kang C, Chen Q, Xia Q, Jiang C, Chen Z, et al. Low-carbon power system dispatch incorporating carbon capture power plants. IEEE Trans Power Syst 2013;28(4):4615–23.